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The bifurcation of channel heads cut by springs

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When groundwater emerges from a spring with sufficient intensity to remove sediment, it channelizes the landscape. Proposed examples of such "seepage channels" include centimeter-scale rills on beaches and levees, hundred-meter-scale valleys on Earth, and kilometer-scale valleys on Mars. Past work has identify the growth and formation of channel heads as responsible for the formation of a network of streams. Here we extend this work by describing a mechanism by which new channel heads form. Based on observations of a kilometer-scale network of seepage channels, we develop a model that describes how the focusing of groundwater into the head causes it either to grow forward or to bifurcate. Rain falling around the network seeps into the ground where variations in the height of the water table cause it to flow into the channels. The resulting subsurface flow is determined by the Poisson equation. Because the size of a stream is very small compared to the spacing between streams, we describe seepage channels as a one-dimensional network growing in a Poisson field. If a random perturbation causes a channel head to bifurcate when it drains water flowing mainly from a single direction, one of the newly bifurcated heads will quickly out-compete the other and a single head will grow forward. In this case, the channel head is stable. Conversely, if a channel head bifurcates when it drains water flowing from all directions, the heads continue to grow in different drainage basins. In this case, the channel head is unstable. We develop a model in which this different behavior is exhibited by a pitchfork bifurcation when the channel reaches a critical length. This bifurcation is a consequence of a difference between the growth of one-dimensional curves in a locally driven, Poisson field versus an infinite Laplace field fed from far away: while a bifurcation in an infinite Laplace field is quickly damped by competition, a Poisson field allows bifurcated heads to remain stable by growing in different directions. This model predicts the critical length of a channel, which we compare to observed bifurcations in the field.