

GALES: a consistent finite element numerical library for the simulation of underground magma dynamics

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The simulation of underground magma dynamics is a challenging, highly non-linear numerical problem. The finite element C++ parallel code GALES, developed to this purpose, solves mass, momentum and energy equations for multiphase multi-component homogeneous gas-liquid (\pm crystals) mixtures with P-T-X-dependent properties. GALES is based on Galerkin-Least Squares Discontinuity Capturing stabilization with space-time discretization. The formulation is stable, robust and accurate, suited for computation of complex dynamics as well as for the solution of several types of conservation equations such as those governing rocks dynamics (under development) or Darcy's flows. Basis functions and the mapping between grid and reference elements depend on space and time, so that mesh deformation is implicit in the method. The use of pressure-primitive variables allows solution for compressible to incompressible flows, since the matrix of change from conservative to primitive variables is well-behaved in the incompressible limit. The stabilizing terms scale with the accuracy of the solution, approaching zero when the solution converges. The Least Squares term adds a streamline weighted transport of information that prevents pure Galerkin solution breakage, reducing and concentrating numerical oscillations in a narrow neighborhood of the internal discontinuities and boundary layers. The Discontinuity Capturing term controls these oscillations. Performed 2D numerical simulations investigate magma dynamics for Stromboli, Etna, and Campi Flegrei magmatic systems characterized by different geometrical complexities and magma composition/properties. The results show complex time-space patterns of magma convection and mixing, previously not described, and oscillatory pressure trends related to the competing effects of buoyant magma rise and dense magma sinking. Integration of the local mass contributions to the gravitational field, and solution of the Green's functions describing pressure wave propagation through the surrounding rock system, provide time-space distribution of ground deformation and gravity anomalies at the Earth's surface and the seismic signals associated with magma motion below the volcanoes. The relationships emerging between deep magma dynamics and recorded geophysical signals lead to better definition of early warning systems and forecasting of the short-term volcanic hazard.