

The stochastic quantization method and its application to the numerical simulation of volcanic conduit dynamics under random conditions

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The use of numerical codes to solve theoretical models of volcanic systems allows a thorough description of the physical and chemical processes occurring at volcanoes. Due to intrinsic uncertainties inherent in the definition of specific volcanic systems and according to the definition of volcanic hazard, the probabilities of possible volcanic scenarios should be estimated as a function of probability distributions describing the input quantities. However, since the computational costs of the numerical codes involved are often very high, it is fundamental to reduce the number of simulations required to reasonably cover the spectrum of possible conditions expected. We accomplish this task through implementation of the Stochastic Quantization (SQ) method [1]. This technique allows optimal approximation of a continuous probability distribution with a discrete one. Our strategy can be applied to general systems with uncertain inputs. We show the results of a benchmark test based on a one-dimensional steady model of multiphase magma flow in volcanic conduits: the output distribution estimated by SQ is very close to a Monte Carlo (MC) estimate of the true one based on 10^3 simulations. In general, the SQ method turns out to provide substantially better results than the MC method with the same number of simulations. Our analysis shows that probabilistic MC scenarios with order 10^3 simulations are well reproduced by order 10 SQ simulations.

Reference

[1] S. Graf, H. Luschgy, Foundations of quantization for probability distributions, Springer, Berlin-Heidelberg, 2000.