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Slepian functions and their use in geophysical signal estimation and spectral analysis

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All geology is local, yet many modern techniques to analyze geophysical signals, from satellite gravity and the geoid, the lithospheric magnetic field, to the making of seismological earth models, heavily rely on a global function basis — the spherical harmonics. When the available data are incomplete and/or noisy, or when the desired models or the features to be extracted are geographically localized (earthquakes, ocean topography, regional-scale anomalies, etc), these have easily demonstrable shortcomings. Enter Slepian functions: firmly rooted in the theory of potential fields, they are designed to combine the best of both the spatial and spectral viewpoints by achieving optimal spatiospectral localization. A basis of Slepian functions for geophysical signals is sparse, and when used as windows to taper data for spectral analysis, they allow for the robust extraction of localized power.

It is a well-known fact that mathematical functions that are timelimited (or spacelimited) cannot be simultaneously bandlimited (in frequency). Yet the finite precision of measurement and computation unavoidably bandlimits our observation and modeling scientific data, and we often only have access to, or are only interested in, a study area that is temporally or spatially bounded. In the geosciences we may be interested in spectrally modeling a time series defined only on a certain interval, or we may want to characterize a specific geographical area observed using an effectively bandlimited measurement device. It is clear that analyzing and representing scientific data of this kind will be facilitated if a basis of functions can be found that are spatiospectrally concentrated, i.e. localized in both domains at the same time. Here, we give a theoretical overview of one particular approach to this concentration problem, as originally proposed for time series by Slepian and coworkers, in the 1960s. We show how this framework leads to practical algorithms and statistically performant methods for the analysis of signals and their power spectra in one and two dimensions, and on the surface of a sphere.

Among the applications that I will discuss are the analysis of the time-variable gravity field for the recovery of coseismic gravity perturbations, the sparse analysis and representation of the lithospheric magnetic field, the recovery of the power spectral density of the cosmic microwave background radiation, and the filtering of the geoid for geodynamic applications.