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Better strategies for finite element solutions of variable viscosity Stokes flow

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The numerical solution of variable viscosity Stokes flow has received relatively little attention by the modeling community until recently. Here we discuss several known computational hurdles to progress, and suggest strategies that offer promise in overcoming them. We will also discuss possibly even more effective MATLAB-based flow-algorithms that can better utilize GPGPUs within a distributed parallel-computing environment.

The choices for solving the discrete pressure equation arising from Stokes flow typically involve several tradeoffs between speed and storage requirements. Our preferred recipe currently has the following ingredients: (1) Use accurate and efficient LBB-stable elements that lead to good convergence in iterative solution schemes (Silvester and Thatcher, 1986; Pelletier et al., 1989). We currently recommend Cornell Macroelements, as they are LBB-stable, are as easy to code as Taylor-Hood elements and have the same convergence properties as them, yet are much more locally incompressible and have no known incompressibility-related artifacts. In contrast, Taylor-Hood elements are 'too squishy' and also have a potential numerical artifact related to capturing an accurate hydrostatic pressure solution (Pelletier et al., 1989). (2) Use an inexact CG pressure preconditioner to drastically reduce the necessary tolerance for the inverse-velocity calculation within it. This leads to a work-reduction of ∼2-3-fold. (3) Use a coarse-grid preconditioner as part of a Dryja-Widlund Additive Schwartz multilevel preconditioner. The key point is to never compute any additional global fine-grid pressure-matrix operations except for the single operation needed to calculate each new CG search direction (i.e. avoid smoothing operations). We are currently exploring an apparently promising > 2 level cascade-type improvement of this general preconditioning strategy.

To make more effective use of GPGPU-enhanced computing we are exploring the use of tile-based DD-like preconditioners and smoothers, and block-adaptations of element-by-element matrix-vector multiplies, which are straightforward to implement in a MATLAB+Jacket programming environment. As a general observation, we find that the relative ease of list-making, book-keeping, vectorizing, and loop-unrolling in MATLAB makes it an extremely useful environment for development of easy-to-enhance parallel finite-element codes for HPC-based research in geodynamics.