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Nonlinear deformation and the elastic energy of damaged rocks

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Crustal rocks are typically treated as linear elastic material with constant elastic moduli. This assumption is appropriate for rocks with relatively low damage, associated with low concentration of cracks and flaws, and under relatively small strains. However, detailed observational results show that rocks and other brittle materials subjected to sufficiently high loads exhibit clear deviations from linear behavior. In general, nonlinear stress-strain relationships of elastic rocks can be approximated by including higher-order terms of the strain tensor in the elastic energy expression (e.g., Murnaghan model). Such models are successful for modeling rock deformation under high confining pressure. However, values of the third (higher order) Murnaghan moduli estimated from acoustic experiments are one to two orders of magnitude above the expected values of the same moduli estimated from the stress-strain relations in quasi-static rock-mechanics experiments. The Murnaghan model also does not reproduce an abrupt change in the elastic moduli when deformation changes from compression to tension. Such behavior is observed in laboratory experiments with rocks, concrete, and composite brittle material samples. Bi-linear models with abrupt changes of the elastic moduli under stress reversal were suggested based on acoustic experiments ("clapping" nonlinearity) and in continuum damage mechanics (unilateral damage model). We present the theoretical basis for general second-order nonlinear expression of the elastic potential. We show that a simplified nonlinear model is consistent with bi-linear elastic behavior and accounts for non-linearity for damaged solids even under small strains. We apply the simplified nonlinear model to various laboratory observations, including quasi-static modeling of composite material with different effective moduli under tension and compression; rock dilation under shear; stress- and damage-induced seismic wave anisotropy observed during cycling load of granite samples; and acoustic experiments with shifts of the resonance frequencies in rock samples. Comparison between analytical and numerical simulations and experimental results demonstrate that the suggested expression for the elastic potential for rocks accounts for both quasi-static damage accumulation and nonlinear dynamic effects.